FREE-JET FORMATION IN HORIZONTAL NON-NEWTONIAN LIQUID PULLING

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A quasi-one-dimensional approach is used to examine the effects of gravitational forces on a horizontally extracted non-Newtonian-liquid jet.

When one-dimensional polymer items are formed horizontally (films or fibers), gravitational forces affect the jets, and the same occurs in rheological research on longitudinal flows [1]. In experiments, the sagging is prevented by pulling in a bath of liquid having nearly the same density. The hydrodynamic friction then needs to be considered. Also, there are substantial difficulties in measuring the tensile stress for a liquid with medium viscosity [1].

The shape of a horizontally pulled jet is dependent in particular on the relation between the gravitational and rheological forces, where research has indicated a new and comparatively simple method of measuring the rheological parameters of polymers during stretching [2]. This can be used in new efficient methods of certifying the drawing parameters of polymer raw materials. Research on longitudinal polymer flow is important [3], and this aspect is of applied interest.

In [4], an equilibrium study was made for a curved liquid jet stretched by its own weight, and a momentum-conservation equation was drawn up for isothermal conditions that incorporated the tension, the inertial forces, and the weight, but the flow scheme discussed there restricted the scope for controlling the pulling.

Here we consider the shape of a free jet pulled under nonisothermal conditions and composed of a non-Newtonian liquid subject to transverse gravitational forces; the flow is steady state in the Euler sense.

1. Formulation. Figure 1 shows the flow scheme. The x and y coordinates characterize the position of the cross-section center. The origin is located where the swelling ceases. The flow terminates at the point where the jet contacts the receiving roll ($x = l$, $y = h$), where the receiving point may be above the horizontal x axis ($h > 0$) or below it ($h < 0$). The pulling zone has a horizontal length l. The point of maximum sag has coordinates x_*, y_*. The axial velocity v is homogeneous over the cross section F. The speed in the swelling section is v_0, while the take-up speed is v_1.

We neglect inertial forces, friction from the surroundings, and surface effects; uniaxial isochoric inhomogeneous stretching occurs [5]. The transverse dimensions are much smaller than the stretching-zone length, $\sqrt{F} \ll l$, and we neglect stresses due to jet bending [6, 7].

The equilibrium conditions [8] imply that the horizontal component H in the tension should be constant (H = const). The tangent is inclined at φ, and the vertical tension component is $H \tan \varphi$. The weight of the jet element ds, whose horizontal projection is dx, is $F\rho g dx/\cos \varphi$.

On the part between x and x + dx, the difference in the vertical tension component is equal to the weight of that jet part, so

$$H \frac{d(\mathrm{tg}\,\varphi)}{dx} = \frac{Q\rho g}{v \cos \varphi}. \tag{1}$$

H is balanced by rheological resistance forces, and the strain-rate tensor $|d|$ in uniaxial flow is

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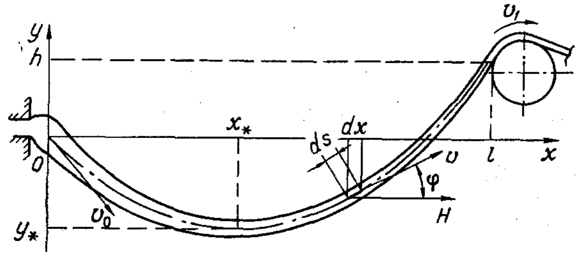

Fig. 1. Free-jet flow scheme.

$$|d| = \frac{dv}{ds}\begin{vmatrix} 1 & 0 & 0 \\ 0 & -0{,}5 & 0 \\ 0 & 0 & -0{,}5 \end{vmatrix}$$

and characterizes the stretching of polymer films [9] and fibers [10]. Here (1) applies for films if $b_0 \ll l$, i.e., pulling a narrow polymer ribbon.

The stress components σ_{ij} in general are

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \quad (i,\ j = 1,\ 2,\ 3). \tag{2}$$

We assume that σ_{11} has the direction of s. The stress tensor deviator components in (2) are $\tau_{ij} = \eta d_{ij}$ (i, j = 1, 2, 3).

We use a semiempirical expression [10] for the material function:

$$\eta = \eta_0 z \left(\frac{I_2}{2}\right)^{\frac{n-1}{2}},$$

where $z = \exp\left[\frac{E}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$.

We use the formulas of [9] for the elongational flow:

$$\sigma_{11} = \eta(d_{11} - d_{22}),\ d_{11} - d_{22} = 3\frac{dv}{ds},\ \frac{I_2}{2} = 3\left(\frac{dv}{ds}\right)^2,$$

and get the axial stress as

$$\sigma_{11} = \eta_0 z (\sqrt{3})^{n+1}\left(\frac{dv}{ds}\right)^n.$$

The horizontal tension component is

$$H = F\sigma_{11}\cos\varphi. \tag{3}$$

As $\tan\varphi = y'$, $\sec\varphi = \sqrt{1+y'^2}$, $ds = dx\sqrt{1+y'^2}$, $F = Q/v$, where the last is derived on the assumption that ρ = const, (1) and (3) become

$$v = \frac{Q\rho g}{Hy''}\sqrt{1+y'^2}, \tag{4}$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{3}}\left(\frac{\rho g}{\sqrt{3}\eta_0 z}\right)^{\frac{1}{n}}\left(\frac{1+y'^2}{y''}\right)^{\frac{1}{n}}\sqrt{1+y'^2}. \tag{5}$$

Here $y' = dy/dx$, $y'' = dy'/dx$.

We take the temperature as uniform over the cross section. The heat transfer with the environment, which has temperature T_c, is described by Newton's law. There are no bulk heat sources. The heat-balance equation for a jet element ds long is [11]

$$\rho c_v Q dT = -\alpha\Pi(T - T_c)\,ds. \tag{6}$$

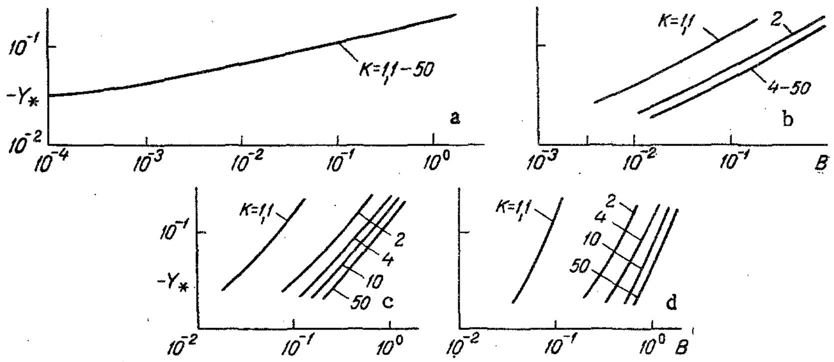

Fig. 2. Dependence of the maximum dimensionless sag on B, K, and n: a) n = 0.2; b) 0.5; c) 1; d) 2. The numbers on the curves are the stretching factors.

The heat-transfer coefficient α may include a radiative component [12].

We introduce dimensionless variables and parameters:

$$X=\frac{x}{l},\ Y=\frac{y}{l},\ V=\frac{v}{v_0},\ \theta=\frac{T-T_c}{T_0-T_c},\ X_*=\frac{x_*}{l},\ Y_*=\frac{y_*}{l},$$

$$K=\frac{v_1}{v_0},\ a=\frac{h}{l},\ A=\frac{H}{\sqrt{3}Q\rho g}\left(\frac{\rho g l}{\sqrt{3}\eta_0}\right)^{\frac{1}{n}},\ B=\frac{l}{\sqrt{3}v_0}\left(\frac{\rho g l}{\sqrt{3}\eta_0}\right)^{\frac{1}{n}}.$$

Then (4), (5), and (6) in dimensionless form are

$$Y''=\frac{B\sqrt{1+Y'^2}}{AV}, \tag{7}$$

$$V'=B\left(\frac{1+Y'^2}{zY''}\right)^{\frac{1}{n}}\sqrt{1+Y'^2}, \tag{8}$$

$$\theta'=-\mathrm{St}\frac{\sqrt{1+Y'^2}}{\sqrt{V}}\theta. \tag{9}$$

Here a prime denotes a derivative with respect to X, $\mathrm{St}=2\alpha b_0 l/\rho c_V Q$ is the Stanton number for a ribbon jet, and $\mathrm{St}=2\pi\alpha l r_0/\rho c_V Q$ is the same for a cylindrical jet.

The boundary conditions are

$$X=0,\ Y=0,\ V=1,\ \theta=1, \tag{10}$$

$$X=1,\ Y=a,\ V=K. \tag{11}$$

At the point of maximum sag,

$$X=X_*,\ Y=Y_*,\ dY/dX=0. \tag{12}$$

2. Isothermal Flow. The solution to (7)-(12) can be represented in quadratures for T = const; as

$$\frac{dV}{dX}=\frac{dV}{dY'}Y'',$$

(8) can be written as

$$\frac{dV}{dY'}=\frac{A^{1+\frac{1}{n}}}{B^{\frac{1}{n}}}V^{1+\frac{1}{n}}(1+Y'^2)^{\frac{1}{2n}}.$$

We separate the variables and integrate on the basis of (10) and (11) to get the dimensionless velocity

$$V=\left[1-\frac{(1-K^{-\frac{1}{n}})}{\beta}\int_{\xi_0}^{\xi}\operatorname{ch}^{\frac{1}{n}+1}\xi d\xi\right]^{-n}. \tag{13}$$

Here

$$\beta=\int_{\xi_0}^{\xi_1}\operatorname{ch}^{\frac{1}{n}+1}\xi d\xi,\ Y'=\operatorname{sh}\xi,\ Y'(X=1)=\operatorname{sh}\xi_1,\ Y'(X=0)=\operatorname{sh}\xi_0.$$

The dimensionless tension is

$$A=\left[\frac{n(1-K^{-\frac{1}{n}})B^{\frac{1}{n}}}{\beta}\right]^{\frac{n}{n+1}}. \tag{14}$$

Equation (7) can be written as

$$\frac{dY'}{dX}=\frac{B\sqrt{1+Y'^2}}{AV}.$$

We separate the variables and integrate to get the longitudinal coordinate

$$X=\frac{A}{B}\int_{\xi_0}^{\xi}Vd\xi. \tag{15}$$

We write (7) as

$$Y'\frac{dY'}{dY}=\frac{B\sqrt{1+Y'^2}}{AV}.$$

We separate the variables and integrate to get

$$Y=\frac{A}{B}\int_{\xi_0}^{\xi}V\operatorname{sh}\xi d\xi. \tag{16}$$

In (13)-(16), ξ is a parameter. The boundary values ξ_0 and ξ_1 are defined by the take-up condition (11). From (15), (16), and (11) we get an integral-equation system for ξ_0 and ξ_1:

$$1=\left[\frac{n(1-K^{-\frac{1}{n}})}{B\beta}\right]^{\frac{n}{n+1}}\int_{\xi_0}^{\xi_1}Vd\xi, \tag{17}$$

$$a=\left[\frac{n(1-K^{-\frac{1}{n}})}{B\beta}\right]^{\frac{n}{n+1}}\int_{\xi_0}^{\xi_1}V\operatorname{sh}\xi d\xi. \tag{18}$$

The maximum-sag coordinates are defined by (12):

$$Y_*=\left[\frac{n(1-K^{-\frac{1}{n}})}{B\beta}\right]^{\frac{n}{n+1}}\int_{\xi_0}^{0}V\operatorname{sh}\xi d\xi, \tag{19}$$

$$X_*=\left[\frac{n(1-K^{-\frac{1}{n}})}{B\beta}\right]^{\frac{n}{n+1}}\int_{\xi_0}^{0}Vd\xi. \tag{20}$$

3. Solution Analysis. Nonisothermal effects, which are characterized by the Stanton number, influence the longitudinal flow in the same way as does any increase in the dilatant parameters [12]. St increase is qualitatively equivalent to increase in n, so the analysis is performed only for the isothermal case. The initial data were:

take-up point at the efflux level $a = 0$; $1.1 \leq K \leq 50$; $0.2 \leq n \leq 2$; $-1 \leq \xi_0 \leq -0.1$. The sequence was that we specified ξ_0 and determined ξ_1 from (18), and then (17) gave B, which was used with ξ_1 in (19) to determine Y_*.

Figure 2 shows that Y_* increases with B and as K decreases. For $n \leq 0.2$, the sag is almost independent of the stretching factor and is governed by B. For a Newtonian liquid (n = 1) or a dilatant one (n > 1), the sag is very much dependent on the stretching factor.

Viscosity-anomaly effects have only a minor influence on the jet shape, as (20) shows (X_* differs slightly from 0.5), and the shape for $a = 0$ is close to parabolic. The axial-velocity distribution is substantially dependent on the take-up point. As a increases, so does the axial-velocity gradient at the take-up point. One can vary a to control the pulling mode and forming conditions within certain limits.

NOTATION

x and y, current coordinates of cross-section center; l, pulling zone length along horizontal; h, take-up point ordinate; x_* and y_*, maximum sag coordinates; F, cross-sectional area; v, axial velocity; v_0 and v_1, initial and final velocities; H, horizontal tension component; φ, tangent slope; s, jet length; ρ, polymer density; g, acceleration due to gravity; Q, volume flow rate; $|d|$, strain rate tensor; σ_{ij}, stress components; p, isotropic pressure; δ_{ij}, Kronecker symbol; τ_{ij}, stress tensor deviator; η, viscosity; I_2, second invariant of strain rate tensor; η_0 and n, rheological constants; z, temperature function; E, activation energy; R, universal gas constant; T_0, initial jet temperature; T, current jet temperature; T_C, environmental temperature; c_V, specific heat; Π, current middle perimeter; α, heat-transfer coefficient; X and Y, dimensionless coordinates; θ, dimensionless temperature; X_* and Y_*, dimensionless maximum-sag coordinates; K, stretching factor; a, A, B, β, ξ_0, ξ_1, dimensionless parameters; St, Stanton number; b_0, initial flat jet width; r_0, initial jet radius; V, dimensionless velocity.

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